METRIC-Bayes: Measurements Estimation for Tracking in High Clutter using Bayesian Nonparametrics

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https://bmoraffa.github.io/presentations

Challenges of Tracking in High Clutter

- Sensor measurements, more often than not, contain detections from false targets.
- Time-dependent number of measurements that include clutter and true sensor observations with unknown origin.
- True measurements from the target are present with some probability of detection.
- Number and location of clutter measurements are random.
- Hence, for accurate tracking, tracking algorithms must only incorporate object generated measurements.

Problem Statement

- Tracking a target in high clutter:
  - State transition equation (first Markovity is assumed)

$$\mathbf{x_k} = \mathbf{f}(\mathbf{x_{k-1}}) + \mathbf{n_k}$$

  - Emission equation:

$$\begin{cases} \mathbf{z^T_{n,k}} = \mathbf{h}(\mathbf{x_k}) + \mathbf{v_{n,k}} & \text{Target originated measurements} \\ \mathbf{z^C_{n,k}} & \text{Clutter measurements} \end{cases}$$

Where $n = 1, 2, \ldots, N_k$, time-dependent!

- If known $\{\mathbf{z_{n,k}}\}_{\mathbf{n=1}}^{\mathbf{N_k}}$, then posterior distribution gives the target state:

$$p(\mathbf{x_k} | \{\mathbf{z_{n,k}}\}_{\mathbf{n=1}}^{\mathbf{N_k}})$$

But we do NOT have the true measurements and presence of clutter deteriorates the performance!

Object Tracking in Clutter: Literature

- Strongest-neighbor and nearest-neighbor filters (NN filter)
  - Measurement that is statistically closest to the predicted measurement is from the object and the rest are clutter!
  - Object motion is linear Gaussian.
  - Disadvantage1: Performance diminishes as probability of false alarm rate increases.
  - Disadvantage2: Uses the philosophy of “winner takes it all”

- Probabilistic data association filter (PDA filter)
  - Used to validate multiple measurements according to their probability of target origin.
  - Assumes object motion obeys linear Gaussian statistics.
  - All non-object originated measurements are assumed to be clutter that is uniformly distributed in the space and Poisson distributed in time.
  - Several variations of the PDA methods proposed:
    - filtered gate structure method; interactive-multiple model PDA; Viterbi and fuzzy data association
  - Disadvantage: PDA type methods can become computationally intensive as the number of measurements increases.

[Li 1998, Kirubarajan 2004, Jeong 2005]

Bayesian Nonparametric Modeling to Rescue!

- Bayesian Statistics:
  - Probabilistic modeling to express all forms of uncertainty and noise
  - … then *inverse probability* rule (i.e. Bayes' Theorem) allows us to infer unknown quantities, learn from data, and make predictions
    - Bayes' theorem:

$$Q(d\theta|X = x) = \frac{dP(X \in \cdot|\theta)}{dP(x \in \cdot)} Q(d\theta)$$

- Bayesian statistics that is not parametric (wait!)
- Bayesian nonparametrics (i.e. not finite parameter, unbounded/ growing/infinite number of parameters)
  - BNP models do not generally satisfy Bayes' theorem since the density cannot exist for all x (undominated models) (not the same as posterior tractability!)
  - Random discrete measures are often undominated.
- Why Bayesian nonparamterics?
  - Bayesian : Simplicity (of the framework)
  - Nonparametric : Complexity (of the real world phenomena)

Bayesian Nonparametrics in Tracking:

- Dependent Dirichlet prior, random infinite tree: time-varying cardinality and evolving states [Moraffah 2018, 2019]
- Hierarchical Dirichlet process prior: prior on unknown number of modes [Fox 2009]
- Bayesian inference: Dirichlet process mixtures for noise in dynamic system [Caron 2008]
- Graphical models for visual object recognition and tracking [Sudderth 2006]
- Learning hierarchical models of scenes, objects, and parts [Sudderth 2005]

Proposed Approach: METRIC Bayes

- Bayesian nonparametric modeling to estimate the measurements that are originated from the target
- METRIC Bayes Intuition:

Model the marginal distributions of the joint prior as two conditionally independent Dirichlet process

- Consider a "joint" Dirichlet process prior over the parameters of true measurements and clutter

$$\mathbf{p}(\mathbf{z}^{\mathbf{T}}, \mathbf{z}^{\mathbf{C}}) = \mathbf{p}(\mathbf{z}^{\mathbf{T}} | \mathbf{z}^{\mathbf{C}})\mathbf{p}(\mathbf{z}^{\mathbf{C}})$$

- Draw parameters associated with each distribution from a Dirichlet process!

[Moraffah 2020]

METRIC Bayes: Prior Distributions

- The hierarchical model describing METRIC Bayes is
  - Prior distributions on clutter parameters at time k:

$$G_k^C \sim \mathbf{DP}(\alpha_{\mathbf{C}}, \mathbf{H_C})$$

$$\theta_{n,k} | G_k^C \sim G_k^C$$

Define: $\Theta_k := \{\theta_{1,k}, \ldots, \theta_{N_k,k}\}$

  - Complete conditional prior on the parameters of true measurements at time k:

$$G_k^T | \Theta_k \sim \mathbf{DP}\Big(\alpha_{\mathbf{T}}, \mathbf{H_T} + \sum_{\mathbf{n=1}}^{\mathbf{N_k}} \delta_{\theta_{\mathbf{n,k}}}\Big)$$

$$\eta_{n,k} | G_k^T \sim G_k^T$$

  - Likelihood distributions:

$$\mathbf{z_{n,k}^T} | \{\eta_{\mathbf{n,k}}\}, \{\theta_{\mathbf{n,k}}\} \sim F_T(\cdot | \{\eta_{n,k}\})$$

$$\mathbf{z_{n,k}^C} | \{\eta_{\mathbf{n,k}}\}, \{\theta_{\mathbf{n,k}}\} \sim F_C(\cdot | \{\theta_{n,k}\})$$

METRIC Bayes: Prior Distributions

- Incorporate true measurements into the Bayesian tracker as follows:
  - Form the likelihood ratio test:

$$\mathcal{L}(\mathbf{z_k^T}; \{\eta_{\mathbf{n,k}}\}, \mathbf{x_k}) = \frac{\prod_{\mathbf{m}} \mathbf{p}(\mathbf{z^T_{m,k}} | \mathbf{x_k}; \text{target present})}{\prod_{\mathbf{m}} \mathbf{p}(\mathbf{z^C_{m,k}} | \mathbf{x_k}; \text{target absent})}$$

- Bayesian tracker:

$$\mathbf{p}(\mathbf{x_k}|\mathbf{z_k}) \propto \underbrace{\mathbf{p}(\mathbf{z_k^T}|\{\eta_{\mathbf{n,k}}\}, \mathbf{x_k})}_{\text{True measurements from likelihood ratio test through METRIC Bayes method}} \underbrace{\mathbf{p}(\mathbf{x_k}|\mathbf{z^T_{k-1}})}_{\text{Bayesian Prediction equation}}$$

True measurements from likelihood ratio test
through METRIC Bayes method

Bayesian Prediction equation

- Sampling using sequential Monte Carlo or Gibbs sampling

(Details in the paper)

METRIC Bayes in one glance

Algorithm 1 METRIC-Bayes: Tracking in high clutter using jointly DPs prior.

Initialize target state: $\mathbf{x}_0$

for $k = 1 : K$ **do**

Predict $p\Big(\mathbf{x}_k \mid \mathbf{Z}^{(\mathrm{t})}_{k-1}\Big)$ using $p\Big(\mathbf{x}_k \mid \mathbf{Z}^{(\mathrm{t})}_{k-1}\Big) = \int p(\mathbf{x}_k \mid \mathbf{x}_{k-1})\, p\Big(\mathbf{x}_{k-1} \mid \mathbf{Z}^{(\mathrm{t})}_{k-1}\Big) d\mathbf{x}_{k-1}$

Input measurements $\{\mathbf{z}_{1,k},\ \ldots,\ \mathbf{z}_{N_k,k}\}$

Draw clutter prior $G^{(\mathrm{c})}_k$ from $\mathrm{DP}(\alpha_c, H_c)$

Use clutter prior to draw $\Theta_k \mid G^{(\mathrm{c})}_k \overset{\mathrm{iid}}{\sim} G^{(\mathrm{c})}_k$

Draw target prior $G^{(\mathrm{t})}_k$ from $G^{(\mathrm{t})}_k \mid \Theta_k \sim \mathrm{DP}\Big(\alpha_t, H_t + \sum_{i=1}^{N_k} \delta_{\theta_{i,k}}\Big)$

Use target prior to draw $W_k \mid G^{(\mathrm{t})}_k, \Theta_k \overset{\mathrm{iid}}{\sim} G^{(\mathrm{t})}_k$

For target measurements, draw $\mathbf{z}^{(\mathrm{t})}_{n,k}$, using Equation (3)

For clutter measurements, draw $\mathbf{z}^{(\mathrm{c})}_{n,k}$, using Equation (4)

Cluster into target measurements $\mathbf{Z}^{(\mathrm{t})}_k$ with cardinality $M^{(\mathrm{t})}_k$

Cluster into clutter measurements $\mathbf{Z}^{(\mathrm{c})}_k$ with cardinality $M^{(\mathrm{c})}_k$

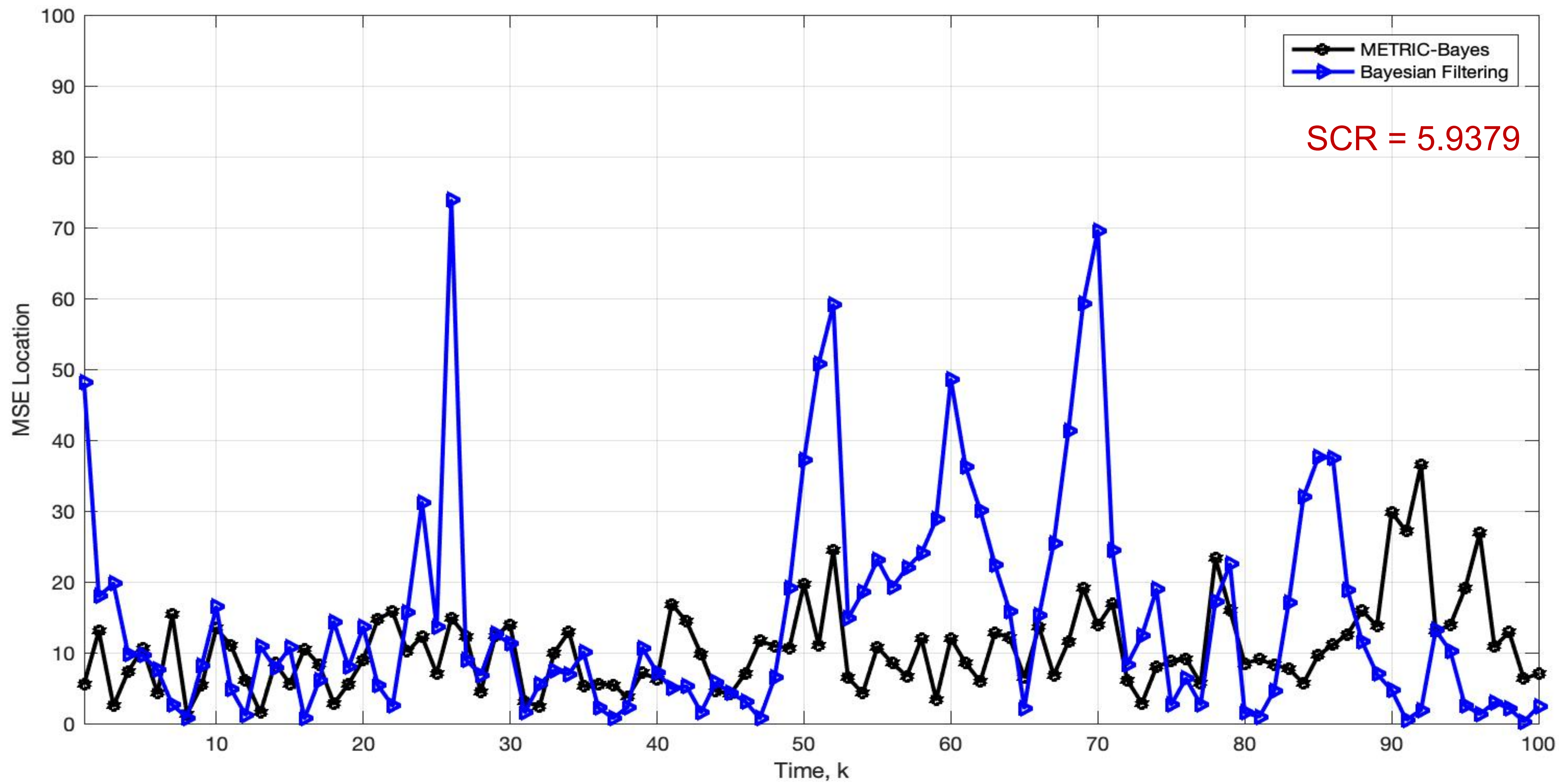
Compute likelihood ratio $L\Big(\mathbf{Z}^{(\mathrm{t})}_k; W_k, \mathbf{x}_k\Big)$ using Equation (5)

Compute and return the posterior density using $p\Big(\mathbf{Z}^{(\mathrm{t})}_k \mid \mathbf{x}_k, W_k\Big)$

Update $p(\mathbf{x}_k \mid \mathbf{Z}^{(\mathrm{t})}_k)$ using $p\Big(\mathbf{x}_k \mid \mathbf{Z}^{(\mathrm{t})}_k\Big) \propto p\Big(\mathbf{Z}^{(\mathrm{t})}_k \mid W_k, \mathbf{x}_k\Big)\, p\Big(\mathbf{x}_k \mid \mathbf{Z}^{(\mathrm{t})}_{k-1}\Big)$

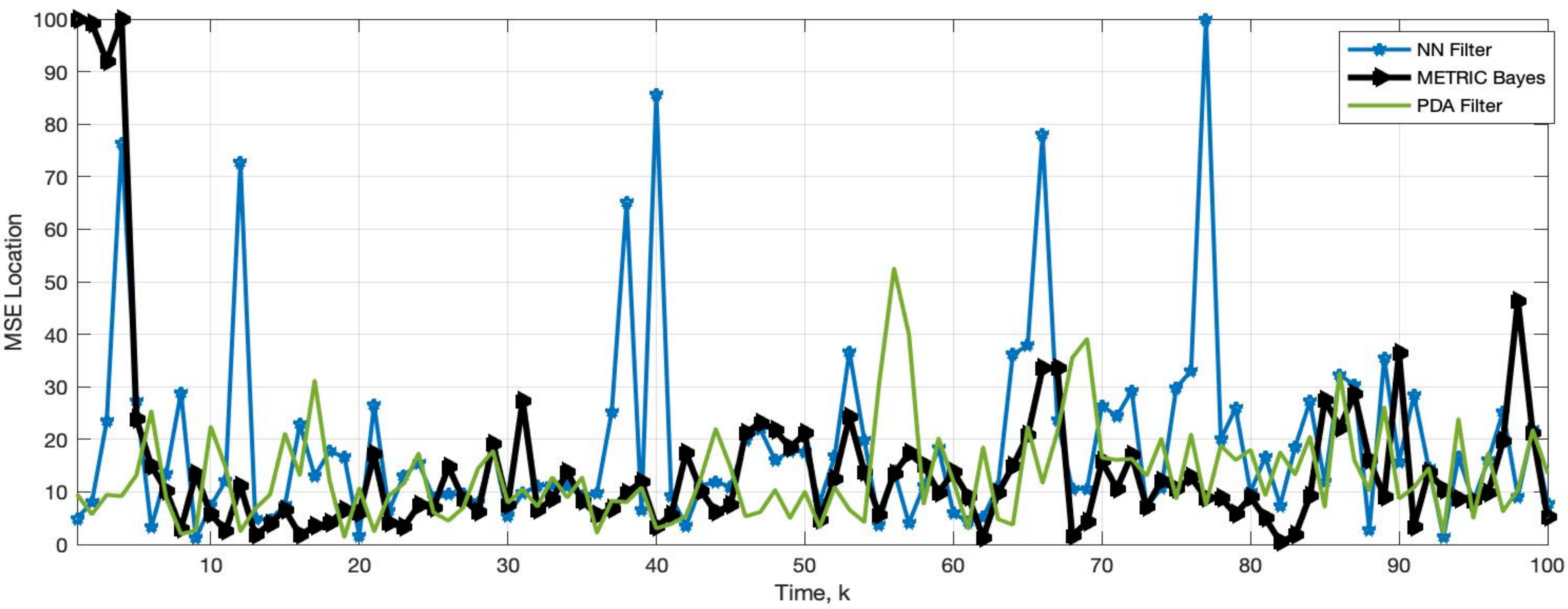
Experiment I: METRIC Bayes vs Bayesian Filtering

Object location estimation mean-squared error (MSE) obtained using METRIC-Bayes vs Bayesian filter that uses all the measurements



Experiment II: METRIC Bayes vs NN and PDF Filters

Object location estimation mean-squared error (MSE) obtained using METRIC-Bayes vs NN and PDA filters for tracking a single object



Conclusions

- Tracking a target in clutter with unknown number of clutters.
- A class of nonparametric models based on a nested joint Dirichlet process
- No assumptions needed for prior knowledge of marginal PDFs.
- Incorporate Bayesian tracker into the modeling.
- Low computational cost as no optimization necessary
- No parametric assumption is made.
- This model can be easily generalized to track multiple objects by incorporating it into a multiple object tracking technique e.g., DDP prior on the states.